

Kaluza-Klein anisotropy in the CMB

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We show that 5-dimensional Kaluza-Klein graviton stresses can slow the decay of shear anisotropy on the brane to observable levels, and we use cosmic microwave background anisotropies to place limits on the initial anisotropy induced by these stresses. An initial shear to Hubble distortion of only $\sim 10^{-3}\Omega_0 h_0^2$ at the 5D Planck time would allow the observed large-angle CMB signal to be a relic mainly of KK tidal effects.

Recent developments in string theory have inspired the construction of braneworld models, in which standard-model fields are confined to our 3-brane universe, while gravity propagates in all the spatial dimensions. A simple 5D class of such models allows for a non-compact extra dimension via a novel mechanism for localization of gravity around the brane at low energies. This mechanism is the warping of the metric by a negative 5D cosmological constant [1]. These models have been generalized to admit cosmological branes [2], and they provide an interesting arena in which to impose cosmological tests on extra-dimensional generalizations of Einstein's theory [3–7].

Modifications to general relativity in the direction of a quantum gravity theory need to be consistent with increasingly detailed cosmological observations. The premier cosmological test is provided by cosmic microwave background (CMB) anisotropies. A detailed calculation of CMB anisotropies predicted by braneworld models is complicated by the need to solve the full 5D perturbation equations, involving partial differential equations for the Fourier modes. Up to now, only qualitative or special results are known [3–7]; in particular, the Sachs-Wolfe effect has not yet been calculated because of 5D effects [5]. The 5D effects are carried by so-called Kaluza-Klein (KK) massive modes of the graviton, which can generate anisotropy on the brane. In view of the great complexity of the full 5D problem, it is worth exploring partial aspects of the problem. In this spirit, we impose some physically reasonable assumptions on the 5D KK effects in order to estimate the CMB large-angle anisotropy. Using COBE observations, this then leads to constraints on the KK anisotropy. We find that the CMB imposes significant limits on the initial anisotropy. It is even possible that the observed large-angle anisotropy derives from anisotropic KK gravitational effects.

Extra-dimensional modifications to Einstein's equations on the brane may be consolidated into an effective total energy-momentum tensor [2,3]:

$$G_{\mu\nu} = \kappa^2 T_{\mu\nu}^{\text{eff}} = \kappa^2 (T_{\mu\nu} + T_{\mu\nu}^{\text{loc}} + T_{\mu\nu}^{\text{kk}}) . \quad (1)$$

Since the brane cosmological constant Λ is negligible at the early times that we consider, we choose the bulk cosmological constant so that $\Lambda = 0$. The local effects of the bulk, arising from the brane extrinsic curvature, are encoded in $T_{\mu\nu}^{\text{loc}} \sim (T_{\mu\nu})^2/\lambda$, and are significant at high energies above the brane tension [4]

$$\rho \gtrsim \lambda \gtrsim 10^8 \text{ GeV}^4 . \quad (2)$$

The nonlocal bulk effects, arising from tidal stresses imprinted on the brane by the bulk Weyl tensor [2,3], are the KK modes carried by $T_{\mu\nu}^{\text{kk}}$. We are interested here in the astrophysically relevant case of small anisotropy and inhomogeneity, and we adopt a long-wavelength velocity-dominated approximation, so that inhomogeneous scalars vary slowly with position and time-derivatives dominate over spatial derivatives. In this approximation, we neglect the acceleration and vorticity of the comoving 4-velocity u^μ . Furthermore, since we are interested in how anisotropic stresses from bulk gravitons affect the shear anisotropy $\sigma_{\mu\nu}$ on the brane, we neglect the matter and KK energy fluxes, and the trace-free anisotropic matter stress. The effective total energy density, pressure and anisotropic stress are therefore [3]

$$\rho^{\text{eff}} = \rho (1 + \rho/2\lambda + \rho^{\text{kk}}/\rho) , \quad (3)$$

$$p^{\text{eff}} = p (1 + \rho/\lambda) + \rho (\rho/2\lambda + \rho^{\text{kk}}/3\rho) , \quad (4)$$

$$\pi_{\mu\nu}^{\text{eff}} = \pi_{\mu\nu}^{\text{kk}} , \quad \pi^{\text{kk}}{}^\mu{}_\mu = 0 . \quad (5)$$

The brane energy-momentum tensor separately satisfies the conservation equations, $\nabla^\nu T_{\mu\nu} = 0$, and the Bianchi identities on the brane imply that the effective energy-momentum tensor is also conserved: $\nabla^\nu T_{\mu\nu}^{\text{eff}} = 0$. In general relativity, anisotropic stresses slow the decay of shear anisotropy [8]. Without anisotropic stress, this slower decay is impossible in general relativity, but in the braneworld, KK graviton stresses can play a similar role to anisotropic stress, as we show below. With our assumptions, the conservation equations (see [3]) reduce to

$$\begin{aligned}\dot{\rho} + \Theta(\rho + p) &= 0, \\ \dot{\rho}^{\text{kk}} + \frac{4}{3}\Theta\rho^{\text{kk}} + \sigma^{\mu\nu}\pi_{\mu\nu}^{\text{kk}} &= 0,\end{aligned}\quad (6)$$

where $\pi_{\mu\nu}^{\text{kk}}$ is transverse as well as tracefree, a dot denotes a comoving time derivative, and $\Theta = 3\dot{a}/a$, where a is an average scale factor.

We also assume that the spatial curvature may be neglected. This leads via the Gauss-Codazzi equations on the brane [9,7] to a shear propagation equation and a Friedmann-like equation:

$$\dot{\sigma}_{\mu\nu} + \Theta\sigma_{\mu\nu} = \pi_{\mu\nu}^{\text{kk}}, \quad (8)$$

$$-\frac{2}{3}\Theta^2 + \sigma^{\mu\nu}\sigma_{\mu\nu} + 2\kappa^2\rho = -\kappa^2\rho^2/\lambda - 2\rho^{\text{kk}}. \quad (9)$$

In Eqs. (6)–(9), there is no evolution equation for the nonlocal KK anisotropic stress $\pi_{\mu\nu}^{\text{kk}}$. This is the anisotropic stress imprinted on the brane by the 5D Weyl tensor, and this nonlocal field cannot be predicted by brane-bound observers since it includes 5D gravitational wave modes. The 5D field equations must be solved to determine the brane dynamics completely, and this also involves choosing boundary conditions in the bulk. On the other hand, starting from a brane-bound viewpoint, any choice of $\pi_{\mu\nu}^{\text{kk}}$ that is consistent with the brane equations, will correspond to a bulk geometry, which can be locally determined in principle by numerical integration (or approximately, close to the brane, by Taylor expanding the Lie-derivative bulk equations given in [2]). However, numerical integration is very complicated (see [10] for the black hole case). Even if it can be successfully performed, it will not give the global properties of the bulk. The bulk geometry that arises for a given $\pi_{\mu\nu}^{\text{kk}}$ may have unphysical boundary conditions or singularities (e.g., the bulk corresponding to a Schwarzschild black hole, with $\rho^{\text{kk}} = \pi_{\mu\nu}^{\text{kk}} = 0$, has a string-like singularity and a singular Cauchy horizon [11]).

We have no exact bulk solutions to guide us in a study of cosmological anisotropy. The only relevant known solution [12,13] is the Schwarzschild-anti de Sitter bulk that contains a (moving) Friedmann brane, which is the exactly isotropic and homogeneous limit of our case, with $\pi_{\mu\nu}^{\text{kk}} = 0$. In the absence of exact or numerical 5D solutions, we are forced to make assumptions about the KK anisotropic stress $\pi_{\mu\nu}^{\text{kk}}$ in order to estimate its impact on the shear anisotropy of the brane. These assumptions should be consistent with the brane equations above, and physically reasonable, and are discussed below.

Observational constraints on the KK stresses arise from big bang nucleosynthesis and from COBE measurements of large-angle CMB temperature anisotropies. The KK energy density on the brane introduces a new radiative degree of freedom at nucleosynthesis. Assuming a maximum of 0.3 of a 2-component neutrino species, and helium limits of 0.228 to 0.248, this gives (compare [12,5])

$$|\rho^{\text{kk}}/\rho|_{\text{ns}} \lesssim .024. \quad (10)$$

The large-angle CMB anisotropies are constrained by

$$\frac{\Delta T}{T} \sim s(t_{\text{ls}}) \lesssim 10^{-5}, \quad s = \sqrt{s_{\mu\nu}s^{\mu\nu}}, \quad s_{\mu\nu} = \frac{\sigma_{\mu\nu}}{\Theta}, \quad (11)$$

where t_{ls} is the time of last scattering. By Eq. (9),

$$s^2 = \frac{2}{3} - 2\kappa^2\rho [1 + \rho/2\lambda + \rho^{\text{kk}}/\rho] / \Theta^2, \quad (12)$$

and then Eq. (10) gives the nucleosynthesis limit

$$s(t_{\text{ns}}) \lesssim 0.13. \quad (13)$$

For small anisotropy, the volume expansion of the universe is determined by the isotropic matter source [we assume that $\rho^{\text{kk}} = 0$ in the background isotropic solution of Eq. (9)]. This amounts to treating a to lowest order as the scale factor for the isotropic Friedmann braneworld. In this approximation, and with equation of state $p = (\gamma - 1)\rho$, with γ constant, Eqs. (6) and (9) lead to

$$\Theta = (2t + \beta)/[\gamma t(t + \beta)], \quad \rho = 4/[3\kappa^2\gamma^2 t(t + \beta)], \quad (14)$$

$$\beta = \sqrt{8/\lambda}/\kappa \lesssim 10^{-9} \text{ sec}, \quad (15)$$

where the bound follows from Eq. (2). This is in agreement with the Friedmann brane solutions given in [12]. (If we take into account the nonlocal energy density in the background solution, i.e. $\rho^{\text{kk}} \neq 0$, we can generalize this solution when $\gamma = \frac{4}{3}$: Θ is still given by Eq. (14), with β replaced by $\tilde{\beta} = \beta[1 + 2\rho^{\text{kk}}/(\kappa^4\lambda\rho)]$, while $\rho \rightarrow (\tilde{\beta}/\beta)\rho$.) The low-energy limit is $\beta \rightarrow 0$, when the general relativity solutions are regained.

Cosmological anisotropy on the brane has been considered in recent papers. A qualitative description of the role of nonlocal anisotropic stress in cosmology is given in [3,7], and perturbative analysis on large scales is developed in [5], but without finding or assuming a form for $\pi_{\mu\nu}^{\text{kk}}$. In [14,15], Bianchi I dynamics on the brane, with vanishing spatial curvature, is studied. In particular, the local extra-dimensional modifications to general relativity introduce a novel feature to early-time dynamics [14]: instead of shear domination, there is *matter* domination at early times, and the relative shear anisotropy s is a maximum when $\rho/\lambda = (2 - \gamma)/(\gamma - 1)$.

Here we investigate the evolution of shear anisotropy in an inhomogeneous universe, at early times (when Λ may be neglected) and in the absence of anisotropic stress from fields on the brane, i.e. $\pi_{\mu\nu} = 0$. We need to determine the effect of KK anisotropic stress $\pi_{\mu\nu}^{\text{kk}}$ on braneworld shear. In principle, the 5D Weyl tensor is determined by the solution of the 5D field equations, and its projection onto the brane then determines $\pi_{\mu\nu}^{\text{kk}}$. In this way, the 5D solution determines an effective evolution equation on the brane for $\pi_{\mu\nu}^{\text{kk}}$. In practice, we know of no way to find or estimate this 5D determination of $\pi_{\mu\nu}^{\text{kk}}$. We emphasize that $\pi_{\mu\nu}^{\text{kk}}$ is a 4D quantity, which is an effective 4D anisotropic stress, even though its evolution is governed by 5D graviton dynamics. Thus it seems reasonable as a first approximation to assume that $\pi_{\mu\nu}^{\text{kk}}$ behaves qualitatively like a general 4D anisotropic stress.

This should be general enough to cover a wide range of bulk graviton effects. Then we can estimate the KK effect on large-angle CMB anisotropies. (Qualitative estimates in the absence of 5D solutions have also been used to estimate CMB anisotropies in different braneworld models, where gravity is modified at large scales rather than high energies [16].)

There is an ansatz that includes all examples of anisotropic stresses studied in relativistic cosmologies and is physically well motivated. According to this ansatz, in the large-scale velocity-dominated approximation, the time evolution of tracefree anisotropic stress is proportional to the energy density of the anisotropic source. This general form includes the known cases of collisionless radiation, (4D) gravitational waves, electric and magnetic fields, strings and walls [8]. We assume that the KK anisotropic stress on the brane behaves qualitatively in a similar way, i.e.,

$$\pi_{\mu\nu}^{\text{kk}} = \rho^{\text{kk}} C_{\mu\nu}, \quad \dot{C}_{\mu\nu} = 0, \quad (16)$$

where $\sqrt{C_{\mu\nu} C^{\mu\nu}}$ is $O(1)$, while ρ^{kk} is perturbatively small relative to the matter energy density ρ , as shown by Eq. (10). The energy conservation equation (7) and the shear evolution equation (9) imply that

$$\dot{r} = (\gamma - \frac{4}{3})\Theta r - \sigma^{\mu\nu} C_{\mu\nu} r, \quad r \equiv \rho^{\text{kk}}/\rho, \quad (17)$$

$$\dot{s}_{\mu\nu} = -(\Theta + \dot{\Theta}/\Theta)s_{\mu\nu} + (\kappa^2 \rho/3H) r C_{\mu\nu}. \quad (18)$$

It is important to notice the special situation that arises in Eq. (17) when the perfect fluid background is radiation ($\gamma = \frac{4}{3}$). In this case the stability of the isotropic solution is determined at *second order*. Linearization about the isotropic expansion would lead to a single zero eigenvalue associated with the shear eigenvalue.

Consider first the simpler case when $\gamma < \frac{4}{3}$. The second term on the right-hand side of Eq. (17) can be neglected with respect to the first and the evolution of the KK energy density is

$$\rho^{\text{kk}} = N(\vec{x})[t(t + \beta)]^{-4/3\gamma}. \quad (19)$$

Although this analysis appears to hold for the models with $\gamma > \frac{4}{3}$, it does not. In these cases $\dot{r} > 0$, and the KK anisotropic stresses would have a gravitational effect that grows with time, invalidating the assumption of Eq. (14) that the volume expansion is well approximated by that of the isotropic solution. The solution for the shear is obtained from the solution of Eqs. (18) and (19):

$$\sigma_{\mu\nu} = [t(t + \beta)]^{-1/\gamma} \left\{ \Sigma_{\mu\nu}(\vec{x}) + \kappa^2 N(\vec{x}) C_{\mu\nu}(\vec{x}) \int [t(t + \beta)]^{-1/3\gamma} dt \right\}. \quad (20)$$

In the low energy regime at late times, $t \gg \beta$,

$$\sigma_{\mu\nu} = \Sigma_{\mu\nu} t^{-2/\gamma} + \frac{3\gamma\kappa^2}{3\gamma - 2} N C_{\mu\nu} t^{-(8-3\gamma)/3\gamma}. \quad (21)$$

This has the same form as the general relativity result when anisotropic matter stresses are present [8]. When the KK anisotropic stress is absent, the solution is determined by the rapidly falling $\Sigma_{\mu\nu}$ mode that is familiar from studies of simple anisotropic Bianchi Type I universes with isotropic stresses. However, when KK anisotropic stresses are present, so that $C_{\mu\nu} \neq 0$, these stresses slow the decay of the shear because of the anisotropic effect of their pressures. The shear evolution becomes increasingly dominated by the $C_{\mu\nu}$ mode at late times. Note in particular that during a dust-dominated ($\gamma = 1$) era the two modes evolve as $\sigma_{\mu\nu} = \Sigma_{\mu\nu} t^{-2} + 3\kappa^2 N C_{\mu\nu} t^{-5/3}$. The KK mode dominates at large t whenever $\gamma > \frac{2}{3}$.

Next, we consider the radiation-dominated solution ($\gamma = \frac{4}{3}$). This is physically the most relevant but is mathematically distinct. The variables Θ and ρ take their isotropic universe values, and Eqs. (17) and (18) have a solution of relaxation form, with $\dot{s}_{\mu\nu} \rightarrow 0$,

$$\kappa^2 \rho^{\text{kk}} = [C_{\alpha\beta}(\vec{x}) C^{\alpha\beta}(\vec{x}) t(t + \tilde{\beta})]^{-1} \times [F(\vec{x}) + \ln(4t^2 + 4\beta t - \beta^2)]^{-1}, \quad (22)$$

$$\sigma_{\mu\nu} = 4\kappa^2 \rho^{\text{kk}}(t, \vec{x}) \left[\frac{t(t + \beta)(2t + \beta)}{4t^2 + 4\beta t - \beta^2} \right] C_{\mu\nu}(\vec{x}). \quad (23)$$

At low energies, $t \gg \beta$,

$$\kappa^2 \rho^{\text{kk}} = [2t^2 C_{\alpha\beta} C^{\alpha\beta} \ln(t/t_*)]^{-1}, \quad (24)$$

$$\sigma_{\mu\nu} = C_{\mu\nu} [t C_{\alpha\beta} C^{\alpha\beta} \ln(t/t_*)]^{-1}, \quad t_* = t_*(\vec{x}), \quad (25)$$

where t_* is some spatially-varying initial time, with $t > t_*$. This has the same form as the general relativity result for matter anisotropic stress [8]. For a diagonal metric with expansion scale factors $a_i(t)$, it leads to $a_i(t) \propto t^{1/2}(\ln t)^{n_i}$, where the constants n_i are of order unity, such that $\Sigma n_i = 0$, and are determined by the eigenvalues of the symmetric tracefree matrix C_{ij} , which specifies the KK anisotropy shape. For example, with an axisymmetric ($a_1 = a_2 \neq a_3$) shear anisotropy, we have $a_1(t) \propto a_2(t) \propto t^{1/2}(\ln t)^{1/4}$ and $a_3(t) \propto t^{1/2}(\ln t)^{-1/2}$.

In the radiation case we note the distinctive slow evolution of the shear anisotropy and non-additive perturbation to the scale factors in the presence of KK anisotropic stress. By Eq. (23), the ratio of shear to Hubble expansion evolves as

$$s \propto \frac{t(t + \beta)}{[F(\vec{x}) + \ln(4t^2 + 4\beta t - \beta^2)](4t^2 + 4\beta t - \beta^2)}, \quad (26)$$

so that at late times it falls only *logarithmically* in time:

$$s \rightarrow 1/[8 \ln(t/t_*)], \quad t \gg \beta > t_*. \quad (27)$$

In the absence of KK anisotropic stresses, s would fall off as $s \propto [t(t + \beta)]^{1/4}/(2t + \beta) \rightarrow t^{-1/2}$. Even if s were $O(1)$ close to the string or Planck scales, it would have become observationally insignificant by the epoch of primordial

nucleosynthesis ($t_{\text{ns}} \sim 1 - 100$ s) or last scattering of the CMB ($t_{\text{ls}} \sim 10^{13}$ s).

The inclusion of the bulk graviton anisotropic stress $\pi_{\mu\nu}^{\text{kk}}$, which has been largely neglected in earlier studies of anisotropic braneworld evolution, completely changes the picture of long-term evolution. This anisotropic stress completely determines the evolution of small expansion anisotropies. Physically, the isotropic stress tends to isotropize the expansion, while the anisotropic stresses tend to resist this isotropizing effect. Since to first order they are both radiation fluids, it is the second-order effect of the KK anisotropy that dominates. Its logarithmic influence reflects its second-order character. In general relativity, this behaviour is only possible if there are anisotropic stresses from matter fields or (4D) gravitational waves. In the braneworld, we can get this behaviour even in the absence of $\pi_{\mu\nu}$, because 5D graviton stresses imprinted on the brane can play the role of $\pi_{\mu\nu}$.

The most interesting feature of the radiation-era solution is that the slow decay of the shear anisotropy allows the shear distortion to have potentially observable consequences. The most sensitive effect is on the CMB rather than nucleosynthesis (this would *not* be the case if the shear decay was a power of time rather than logarithmic). If gravity in the bulk induces an initial anisotropic stress on the brane, $s(t_{\text{in}})$, then during the radiation-dominated era, $s \propto [\ln(t/t_*)]^{-1}$, with $t_*(\vec{x})$ time-independent, by Eq. (27). During the short interval of dust-dominated evolution from the equal-density epoch t_{eq} until t_{ls} , we have $s \propto t^{-2/3}$ by Eq. (21). Thus the CMB temperature anisotropy on large angular scales will be

$$\frac{\Delta T}{T} \sim s(t_{\text{ls}}) \approx s(t_{\text{in}}) \left(\frac{t_{\text{eq}}}{t_{\text{ls}}} \right)^{2/3} \left[\frac{\ln(t_{\text{in}}/t_*)}{\ln(t_{\text{eq}}/t_*)} \right]. \quad (28)$$

Crucially, the magnitude of $\Delta T/T$ is only logarithmically dependent on t_{in} and t_* . Using $1 + z_{\text{ls}} = 1100$ and $1 + z_{\text{eq}} = 2.4 \times 10^4 \Omega_0 h_0^2$, where Ω_0 is the present total matter density of the universe in units of the critical density and h_0 is the Hubble parameter today in units of $100 \text{ km s}^{-1} \text{ Mpc}^{-1}$, we find

$$\frac{\Delta T}{T} \sim 4.6 \times 10^{-2} \Omega_0^{-1} h_0^{-2} s(t_{\text{in}}) \left[\frac{\ln(t_{\text{in}}/t_*)}{\ln(t_{\text{eq}}/t_*)} \right]. \quad (29)$$

The time t_{in} could reasonably be taken as the 5D Planck time t_5 . By Eq. (2) the 5D Planck mass is subject to $M_5 \gtrsim 10^8 \text{ GeV}$, so that $t_5 \lesssim 10^{11} t_4$, where $t_4 \approx 10^{-43} \text{ sec}$ is the 4D Planck time. For $t_* \sim t_4 \approx 10^{-43} \text{ sec}$ and $t_{\text{in}} \sim 10^{10} t_*$, the logarithmic term would be $\sim \frac{1}{5}$, and is relatively insensitive to quite large changes in these quantities. Thus, using Eq. (11),

$$s(t_{\text{in}}) \lesssim 10^{-3} \Omega_0 h_0^2. \quad (30)$$

This is a much tighter constraint on the initial anisotropy than obtained from nucleosynthesis, Eq. (13). The observed large-angle temperature anisotropy in the CMB

may have been contributed by bulk graviton effects in the very early universe if they have an initial amplitude of $\sim 10^{-3} \Omega_0 h_0^2$. This anisotropy level is too low to have an observable effect on the output from primordial nucleosynthesis.

We have shown that the tidal stresses induced on a perfect-fluid braneworld by bulk gravitons are the dominant factor in determining the evolution of its anisotropic distortion. During the radiation era this distortion falls only logarithmically in time relative to the expansion rate of the brane and can contribute a significant component to the large angular scale anisotropy of the CMB. An initial σ/Θ ratio of only $s(t_{\text{in}}) \sim 10^{-3} \Omega_0 h_0^2$ would allow the observed CMB signal to be a relic mainly of KK graviton tidal effects.

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